[2] Howe, J. T., Green, M. J., and Weston, K. C., "Thermal Shielding by Subliming Volume Reflectors in Convective and Intense Radiative Environments," *AIAA Journal*, Vol. 11, No. 7, July 1973, pp. 989–994.

[3] Reddy, R. S., "Approximate Analyses for the Transient Thermal Response of a Weakly Absorbing Medium Undergoing Combined Conductive and Radiative Transfer," Masters thesis, 1973, Univ. of Tulsa, Tulsa, Okla.

Verification of a Simple Relationship for Shock-Wave Reflection in a Relaxing Gas

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A SIMPLIFIED description of shock-wave reflection in a relaxing gas was recently presented by Hanson.[1] The theoretical model, which has useful applications to shock-tube studies, is based on the fact that the characteristic relaxation time behind a reflected shock wave is generally much shorter than the corresponding relaxation time behind the preceding incident shock wave. A useful result of the approximate theory is the simple relationship between the time-varying pressure at a shock-tube end wall and the spatially varying density in the incident-shock relaxation zone[1]

$$P(t)/P_{\text{final}} = [\eta(s)-1]/(\eta_{\text{final}}-1)$$

The function η is the spatially varying density ratio ρ/ρ_1, evaluated at the distance s behind the incident shock wave, and t is the time after shock reflection. Thus measurements of pressure can be used to study incident-shock relaxation phenomena, without requiring complex calculations of the shock-reflection process and with significant possible increases in accuracy, operating range and temporal resolution over density measuring techniques.

The pressure measurement technique has already been successfully applied in several studies of vibrational[2,3] and chemical[4,5] kinetics, although the accuracy of the simple relationship between pressure and density has not been confirmed by direct experiment. The present study provides such direct experimental verification in the form of simultaneous measurements of density and pressure in the same shock-wave relaxation experiment.

The experiments were conducted in vibrationally relaxing O_2. The density in the incident-shock relaxation zone was monitored using a quantitative laser-schlieren method similar to that developed by Kiefer and Lutz[6] and described recently by Breshears, Bird, and Kiefer.[7] The method involves measurement of the deflection of a laser beam due to passage of the beam through the relaxing gas behind a shock wave. The relaxation produces an axial density gradient $d\rho/dy$ within the shock tube, and this density gradient causes a beam deflection D at a distance L beyond the shock tube of

$$D = RLw(d\rho/dy)$$

where R is the specific refractivity of the gas (0.19 cm^3/gm for O_2) and w is the internal width of the shock tube.

A schematic of the present arrangement is shown in Fig. 1. The 6328 A beam from a 2 mw helium-neon laser (Spectra-Physics Model 133) is reduced in diameter (to about 0.3 mm) with a simple telescope, and passed through the shock tube in a plane normal to the tube axis. The beam is tilted slightly in the vertical direction to avoid interference between the main beam and components reflected from the window surfaces. Two meters from the center of the tube the beam is divided by a metal-coated prism, and the individual beams are focussed onto matching photodiode detectors (RCA C30808, 2.5-mm-diam active area). The entire optical system is compactly arranged on a single 60 cm × 120 cm vibration-isolated table.

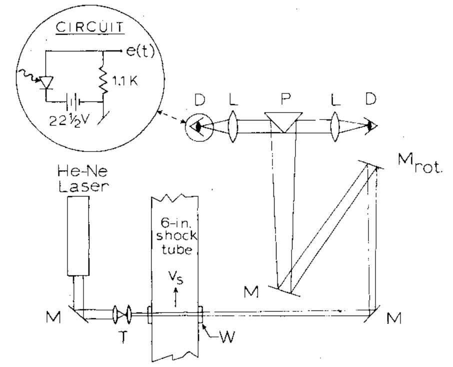

Fig. 1 Laser-schlieren system for incident-shock measurements of density gradient: M = mirror, P = prism (metal-coated), W = window (8 mm dia), L = lens (10-cm focal length), D = photodiode detector, T = telescope (2.7 power).

Deflection of the beam from its initially centered position on the prism apex produces equal and opposite changes in light intensity at the detectors, which are a.c.-coupled to the differential preamplifier of an oscilloscope. Differencing the detector outputs thus provides a signal proportional to beam deflection, but with reduced noise and doubled sensitivity over a single-beam system. For small beam deflections, the voltage displayed on the oscilloscope is linearly proportional to beam deflection and hence also to the density gradient.[6,7] The overall detector circuit-oscilloscope response time (determined by the load resistor and the circuit capacitance to ground) is less than 0.1 μsec (0–63%).

The system was calibrated using a rotating mirror (M_{rot} in Fig. 1) mounted on the shaft of a small electric motor. Knowledge of the rotational speed (3300 rpm) and distance from mirror to prism are sufficient to infer a calibration constant (about 250 mv/mm of deflection at the prism) from an oscillogram. Removal of the telescope from this same optical arrangement results in a reduction in spatial resolution and an increase in sensitivity of about a factor of two.

Results obtained with the laser-schlieren system for an $M = 5.51$ shock wave into O_2 (taken directly from a commercial cylinder, purity greater than 99.99%) are shown in Fig. 2. For comparison,

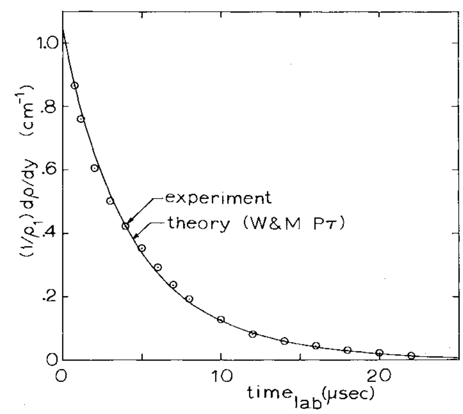

Fig. 2 Reduced record of the density gradient for a shock wave in pure O_2. The experimental conditions were: P_1 = 4.33 torr, V_s = 1.817 mm/μsec, T_1 = 299°K. The solid curve was calculated using the relaxation-time expression of White and Millikan.[8]

Received June 7, 1973. This research was supported by NASA Grant NGR-05-020-583.

Index categories: Reactive Flows; Shock Waves and Detonations.

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a theoretical prediction of the density gradient also is shown. This theoretical result was obtained by numerically integrating the Bethe-Teller vibrational relaxation equation subject to the shock-wave conservation relations and with the relaxation-time expression suggested by White and Millikan.[8] The agreement between experiment and theory is quite good, the maximum discrepancy being about equal to the experimental uncertainty (associated principally with the uncertainties in system calibration and in the recording and transfer of data using oscillograms). As suggested by Kiefer et al., zero time was taken at the peak in the voltage signal, in this case at 0.5 μsec after the initial transient. An effective lower limit on the spatial resolution near the shock front for this experiment is therefore about 0.9 mm. Experiments at higher pressure with less shock-wave curvature would permit resolution approaching the halfwidth of the laser beam.

Experimental results for the end-wall pressure, obtained with a Baganoff capacitance-type pressure gauge,[9] are presented in normalized form in Fig. 3. Also shown are two theoretical wall-pressure time histories calculated using the simple relation for pressure and density under test here. The solid curve is based on the density profile $\eta(s)$ predicted by White and Millikan's expression for relaxation time ($P\tau$), and the cross (+) points were computed with the density profile obtained by integrating the laser-schlieren data shown in Fig. 2. The density gradient was integrated backward in time from vibrational equilibrium, thereby yielding $\eta(s)$ since η_{final} is known from the initial conditions and the measured shock speed.

The prescription for mapping the distance s into the time after shock reflection t, derivable from simple geometrical arguments,[1,3] is

$$t = (s/V_s)[(1 + V_r/a_5)/(1 + V_r/V_s)]$$

where V_s and V_r are, respectively, the incident- and reflected-shock speeds, and a_5 is the equilibrium speed of sound in the reflected-shock region. Both V_r and a_5 are evaluated using the Rankine-Hugoniot shock-jump relations assuming vibrational equilibrium upstream and downstream of the reflected shock wave.[1]

The good agreement between measured and calculated pressure histories is taken as verification, within the experimental undertainty, of the simple relationship between pressure and density. (It is worth noting that the slight decrement in measured pressure visible for small t is expected due to the known effect of heat conduction to the shock-tube end wall.[10]) The close agreement between the two theoretical pressure-time histories follows from the agreement previously shown in Fig. 2, since the same pressure-density relationship was applied in both cases.

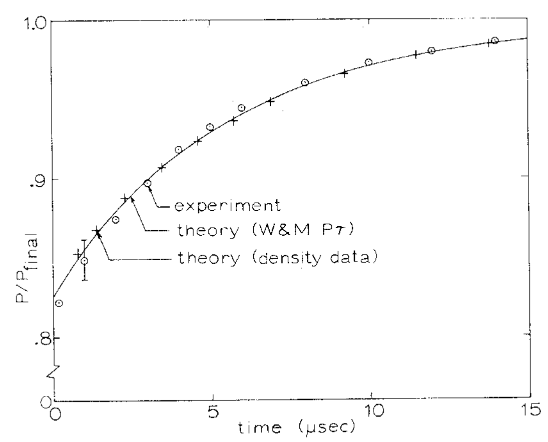

Fig. 3 Record of the end-wall pressure history for a shock wave in pure O_2. The experimental conditions were: $P_1 = 4.33$ torr, $V_s = 1.817$ mm/μsec, $T_1 = 299$°K. The theoretical results were calculated using the simple theory relating pressure and density for shock-wave reflection: —— based on the relaxation time expression of White and Millikan[8]; +, based on an integration of the laser-schlieren data in Fig. 2.

The present study, of course, also serves to further substantiate the relaxation-time expression for O_2 suggested by White and Millikan.

References

[1] Hanson, R. K., "Shock-Wave Reflection in a Relaxing Gas," *Journal of Fluid Mechanics*, Vol. 45, Pt. 4, Feb. 1971, pp. 721–746.

[2] Hanson, R. K. and Baganoff, D., "Shock-Tube Study of Vibrational Relaxation in Nitrogen Using Pressure Measurements," *Journal of Chemical Physics*, Vol. 53, No. 11, Dec. 1970, pp. 4401–4403.

[3] Hanson, R. K., "Shock-Tube Study of Vibrational Relaxation in Carbon Monoxide Using Pressure Measurements," *AIAA Journal*, Vol. 9, No. 9, Sept. 1971, pp. 1811–1819.

[4] Hanson, R. K. and Baganoff, D., "Shock-Tube Study of Nitrogen Dissociation Rates Using Pressure Measurements," *AIAA Journal*, Vol. 10, No. 2, Feb. 1972, pp. 211–215.

[5] Hanson, R. K., "Shock-Tube Study of Carbon Monoxide Dissociation Kinetics," *Journal of Chemical Physics*, Dec. 1973.

[6] Kiefer, J. H. and Lutz, R. W., "Vibrational Relaxation of Deuterium by a Quantitative Schlieren Method," *Journal of Chemical Physics*, Vol. 44, No. 2, Jan. 1966, pp. 658–667.

[7] Breshears, W. D., Bird, P. F., and Kiefer, J. H., "Density Gradient Measurements of O_2 Dissociation in Shock Waves," *Journal of Chemical Physics*, Vol. 55, No. 8, Oct. 1971, pp. 4017–4026.

[8] White, D. R. and Millikan, R. C., "Vibrational Relaxation of Oxygen," *Journal of Chemical Physics*, Vol. 39, No. 7, Oct. 1963, pp. 1803–1806.

[9] Hanson, R. K. and Baganoff, D., "Improved Fast-Response Pressure Gauge for Shock-Reflection Studies in Ionized Gases," *Review of Scientific Instruments*, Vol. 43, No. 3, March 1972, pp. 394–399.

[10] Baganoff, D., "Experiments on the Wall-Pressure History in Shock-Reflexion Processes," *Journal of Fluid Mechanics*, Vol. 28, Pt. 2, 1965, pp. 209–228.

Integral Equation for Small Perturbations of Irrotational Flows

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Introduction

The problem of computing the effects of thickness on aerodynamic forces acting on oscillating wings and bodies, especially in the transonic speed range, has attracted considerable interest. The aim of this Note is to explore the validity of analytical methods which are based on deriving an integral equation from the application of small perturbations to a nonuniform but irrotational flow, such as might approximate the flow over a thick wing or body. Thus, the following observations apply to a wide Mach number range, but are restricted to small amplitude motions, such as at the onset of flutter, and to nonviscous flows.

It is shown that the integral equation relating the unknown velocity potential to the known normal flow velocity can be derived from the appropriate Green's identity. However, the Green's function from which the kernel of this integral equation is formed is not the unit source solution, but the unit solution

Received June 7, 1973. This work was carried out under a grant from the NASA Langley Laboratories.

Index category: Nonsteady Aerodynamics.

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